

Robust wavelet estimation and quality measures

Peter Harris*, Rock Solid Images AS, Oslo.

Summary

In this paper we describe a stochastic method of wavelet estimation tying well logs to seismic data. It has the advantage over traditional methods that it is statistically robust, that is, it is insensitive to outliers in the reflection coefficients and in the seismic data. This is achieved by generating random perturbations to an initial wavelet and comparing the resulting synthetics to the seismic data by means of a robust norm. A set of best-matching wavelets is determined rather than a single solution, thus allowing us to attach uncertainty estimates to the wavelet and the synthetics. A further refinement is to use an iterative process similar to Huber regression in determining the best-matching wavelets. This requires weighting the residuals by a function of the misfit determined in the previous iteration, thereby downweighting those parts of the data where the seismic data and synthetic are in poor agreement.

Perhaps more important than the wavelets themselves are the resulting quality measures. Uncertainty in the amplitude and phase spectra of the wavelets are obtained. Wavelets from multiple wells within an area can then be compared quantitatively to ensure that the seismic phase is consistent over the area before starting an inversion. The weights are also useful in indicating where there is a mismatch between logs and seismic data. Such mismatches may be due to problems in the logs, such as poor log editing, or to seismic problems such as residual multiples. In either case the weights show where the data need further processing before starting reservoir characterisation work.

Introduction

There is an increasing need in reservoir characterisation studies to associate uncertainty estimates with any parameters of interest obtained from seismic data. For the most part, such parameters require calibration at wells and therefore the accuracy of the tie between log data and the seismic data is the key to accurate calibration. We therefore view the process of well tie as the comparison or integration of two different sets of physical measurements, the logs and the seismic data. Although this paper describes a method of wavelet estimation, perhaps the most important result from our analysis is the measures indicating the quality of the tie. These measures help to show where there are problems in either logs or seismic data, and the uncertainty estimates from the tie feed into the uncertainties of derived reservoir properties.

The assessment of well tie quality by partial coherence analysis was clearly expounded by R.E White in 1980. The

emphasis on quality measures in the present work is intended to build on that by exploiting robust statistical methods. We first describe the wavelet estimation procedure, and then discuss the quality measures and some ways in which they are exploited.

Robust wavelet estimation

Classical estimation of a wavelet from comparison of logs and seismic data involves setting up a set of linear equations relating the reflection coefficients to the seismic trace. Typically these are solved by a least-squares method, and we call the solution our estimate of the wavelet. The underlying statistical assumptions are that the reflection coefficients contain no noise, and that any errors or noise in the seismic data is Gaussian.

The second of these assumptions, that the seismic noise is Gaussian, may be reasonable. However the first, that reflection coefficients contain no errors, is evidently wrong since they are derived from physical measurements. Furthermore, the largest errors in the reflection coefficients are often the consequence of washouts, cycle skipping, or other localised defects in the well logs. Such errors are generally large and far from Gaussian, although they may have quite localised extent in depth.

The method of total least squares (eg Golub and van Loan, 1980) was developed to solve linear equations in the presence of errors in both dependent and independent variables. However, it is not well-suited to the present problem because the assumption remains that errors are Gaussian. Here we develop a robust algorithm related to Huber regression (Huber, 1981) which is less sensitive to the presence of outliers in the reflection coefficients.

The algorithm works by generating perturbations to an initial estimate of the wavelet and convolving the resulting new wavelet with the reflectivity series to make a synthetic. This is compared with the seismic trace to obtain a misfit value:

$$\varepsilon = \sum_j h_j f(y_j - w * r_j) \quad (1)$$

where y is the seismic trace, w is the wavelet, r is the reflectivity series, h is a set of weights, and the asterisk denotes convolution. The sum is taken over all the samples within a time window of interest. The function f may be chosen according to the assumed distribution of the errors. To make the algorithm robust, we use the absolute value for f . Rather than seeking a single wavelet minimising ε , we save all wavelets for which ε falls below some threshold. They are then analysed to give uncertainty estimates for the wavelet amplitude and phase spectra.

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In the first pass of the algorithm, the weights are set to be equal. After producing a set of “good” wavelets, the corresponding synthetics are compared with the seismic trace to produce a sample-by-sample misfit function. A new set of weights is defined from the misfit function, and a second iteration is performed of wavelet perturbation, synthetic generation, and selection of the “best” wavelets according to equation (1) with the new weights. One might continue with more iterations, but we have not found any substantial improvement in the results after two iterations.

Figure 1 shows the misfit values of the best 50 wavelets from a run in which 20000 wavelets were generated. The best 10 wavelets are shown superimposed in figure 2. It can be seen that the wavelet tails are less stable than the main lobes. This means that weak events close to stronger ones are likely to suffer from a high degree of uncertainty due to this wavelet instability.

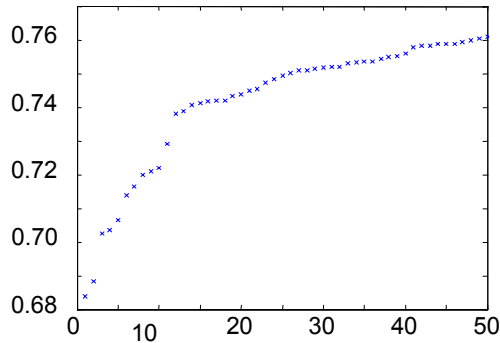


Figure 1: Values of the misfit function (eq 1) for the best 50 wavelets of 20000 tested. The first 10 have lower values than the rest, and they are selected for further analysis.

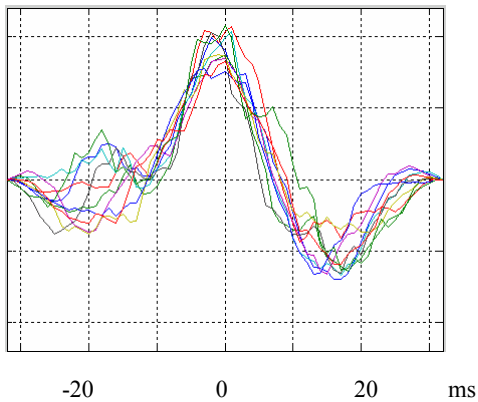


Figure 2: The 10 best wavelets

The effect of the weights is to emphasise those parts of the data where the seismic-synthetic tie is good while downweighting areas of mismatch. The main benefit is that the wavelet phase is estimated more stably in the second iteration (fig 3). To obtain a final single wavelet, we apply principal component analysis to the set of good wavelets, a method suggested by Tury Taner. It works well provided that the starting wavelets are essentially in phase, as is the case here.

Quality measures

The use of a stochastic algorithm allows us to explore the uncertainty in the estimated wavelet. Uncertainty in the phase affects event timing, with consequences for volumetric estimates. Quantitative comparison of wavelets extracted at different wells within a prospect allows one to determine whether the wavelets differ significantly or, alternatively, whether variations between them are within the level of uncertainty and may therefore be discounted.

The weights obtained from the fitting algorithm may be used to indicate where there is a mismatch between seismic and well data (figs 5 & 6). There are several ways of exploiting this information. Scanning a small cube of traces around the nominal well position is used to identify relative positioning errors between well and seismic. Mismatches in short time intervals may indicate problems either in the seismic data or in the logs, for example residual multiples or inadequate log editing.

The misfit value of equation (1) can also be exploited. Figure 4 shows a seismic trace and synthetic obtained from the best fitting wavelet. Despite careful checkshot editing the tie is poor, with few events corresponding in time. We applied linear compression to the time axis of the reflection coefficients and chose the compression rate of 0.94 which minimised the misfit (table 1). After applying the compression, the tie is much improved (fig 5) and most events can be identified on both seismic and synthetic. A further improvement would be obtained by splitting the data into shorter windows and calculating different compression rates for each.

We believe that the compression may be due to (VTI) anisotropy. The rate of 0.94 corresponds to Thomsen's δ taking a value of 0.066, which is high but certainly possible in shales. By contrast, Backus averaging the logs at 1/10 of the minimum wavelength with the assumption of isotropic layering at the log scale results in values of δ around -0.01, much too small in magnitude and the wrong sign to predict the observations. Thus we have to conclude that, if anisotropy is indeed the explanation, it is occurring at a finer scale than the log sampling.

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Conclusion

In reservoir characterisation work the first key requirement is to obtain a good time-depth relationship. In anisotropic depth imaging well tie is necessary to estimate δ , which cannot be estimated from seismic data alone. In inversion studies, a stable wavelet and spatially consistent seismic phase are necessary for a meaningful result. Quantitative analysis of time-lapse seismic also requires calibration at wells. For all these and many more purposes, quantitative measures of quality are essential both to aid in decision making and to estimate uncertainty.

This paper describes one approach to obtaining those quality measures. Clearly it is not the only one, but it is seen to be robust and capable of producing more useful information than would be obtained from, say, a simple correlation coefficient.

Acknowledgements

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References

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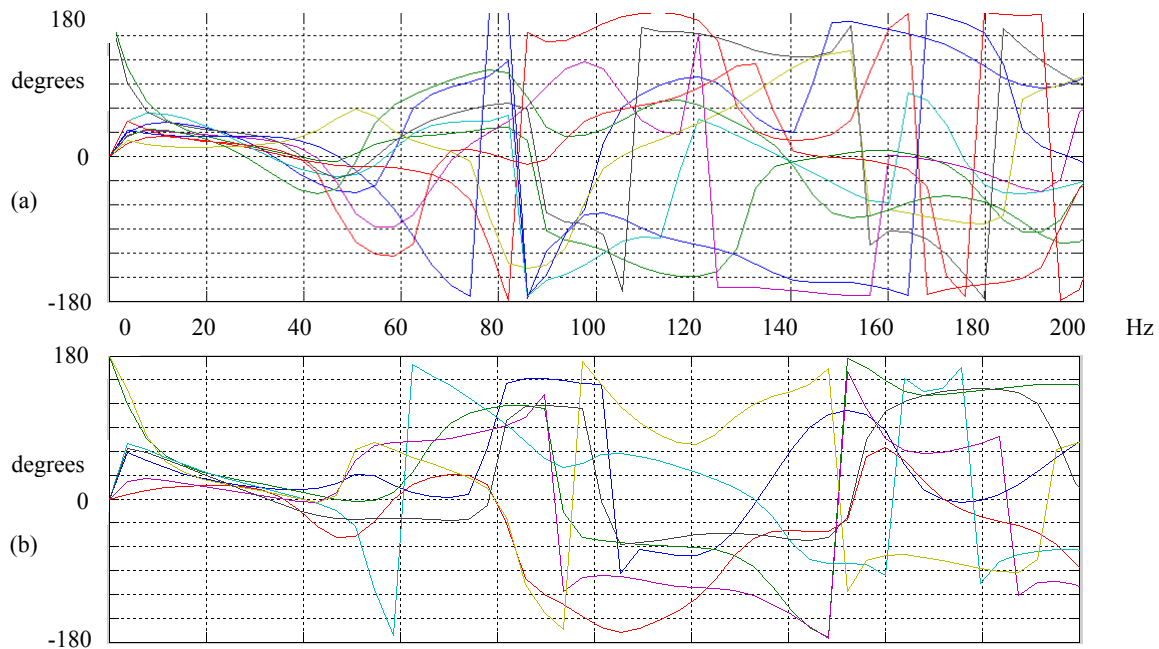


Figure 3: Phase spectra of the set of best wavelets obtained from the first, unweighted iteration (a) and the second (weighted) iteration. The phase within the seismic bandwidth is more stable in the second iteration.

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Compression rate	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
Lowest misfit value	1.315	1.170	1.083	1.049	1.056	1.159	1.173	1.250	1.309

Table 1: Values of the misfit function for different compression rates of the reflection coefficients time axis.

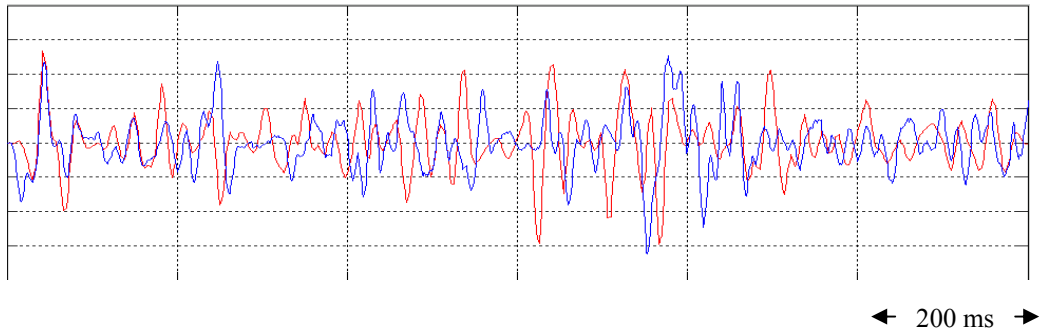


Figure 4: Seismic trace (red) at the well location and synthetic trace (blue). The first main event ties, but later it is hard to identify corresponding events. TVD and checkshot corrections have been applied.

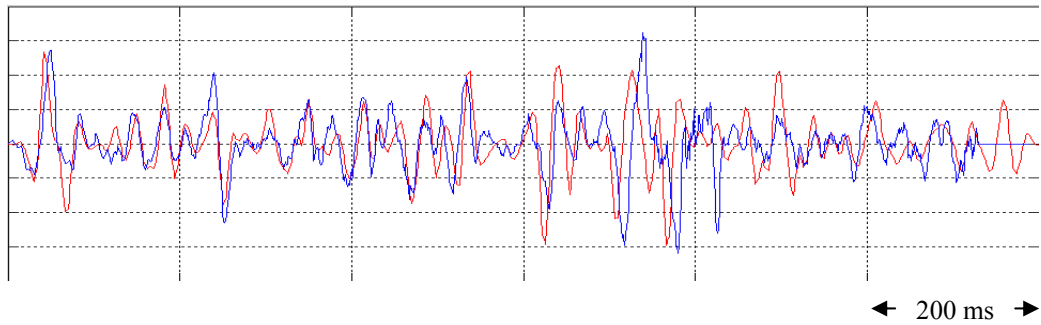


Figure 5: Seismic trace (red) at the well location and synthetic trace (blue) after the time axis of the reflection coefficients has been compressed by 0.94. Most events are now identifiable, even where the tie is not perfect.

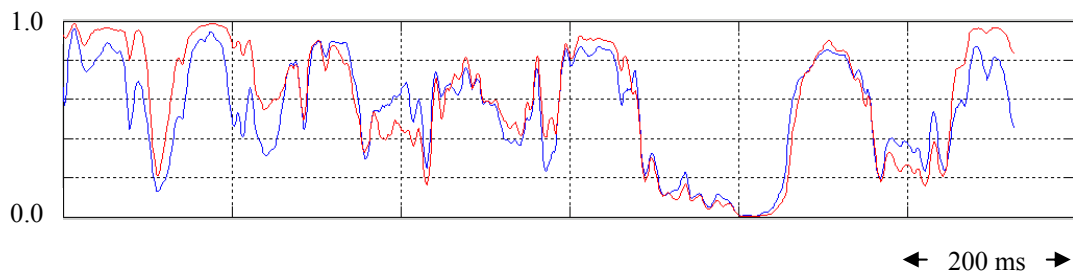


Figure 6: Weights obtained after the first unweighted pass (blue) and the second weighted pass (red). They are low where there is a residual apparent time shift between the seismic trace and the synthetic.