# MULTI-COMPONENT
# EDGE DETECTION ALGORITHM

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October 1979, Revised April 1999

**INTRODUCTION:**

In picture processing, one of the most effective procedures to sharpen the images is to improve the contrast. The contrast is improved by increasing the difference across discontinuities of the image components. In order to improve the differences, we have to detect them. The Edge Detection algorithm is designed to detect and highlight these discontinuities. It was first developed for processing satellite pictures. The technique has become well known and used widely in digital image processing. In seismic interpretation, the edges come in many forms and shapes. They are in the form of faults, stratigraphic boundaries, limits of production, etc. Therefore, a good edge detection procedure is a valuable tool, which should be included in the interpretation kit.
I have given a number of references below. The most recent and beneficial ones are those co-authored by Dessing in the Delphi reports and his Ph. D. thesis.
The edge detection algorithm can also be used to detect various stratigraphic or lithological boundaries, as well as its conventional use in the image contrast enhancement. Each seismic data sample is represented by a number of attributes, amplitude, phase, dip, azimuth, frequency and etc. In the case of picture processing we detect sharp changes of the recorded light energy, which is only a single component detection. Since seismic data can be presented as a multi-component vector, we can apply the same paradigm to detect various types of discontinuities. For example, if we use essentially the geometrical attributes, we should detect discontinuities of bedding patterns. If we select lithology related attributes, the edges we detect will be more related to the edges of various lithologies. In this report we will describe a generalized multi-component edge detection algorithm which can be used for more purposes than just fault detection.

**METHOD:**

The edge detection algorithm is a generalization of the dip computation. Dips are determined by computing the partial differences in ***x,y*** and ***t*** directions. This corresponds to computing the differences on the high end of the frequency spectrum. The generalized method exploits the wavelet decomposition approach to enhance and detect edges over various frequency bands. One of the earliest uses of edge detection was in the first break picking. This was done by computing the power ratio of two running time windows and picking the maximum, which corresponds to the boundary between the pre-first break noise zone and the energetic first break arrivals. Using only the power ratio gave a number of mis-picks. This defect was improved by including several other attributes. In the conventional method, the difference between the two time windows is computed, from which the edges are detected. This corresponds to the band limited difference. There are a number of articles on the edge detection algorithms and their variations. I have found an excellent review in the Delphi Consortium report. In this report there are several references to the work of Mallat and Zhong (1992). I will follow the development given in the Delphi report.
To describe the method simply, we will first discuss the general single component detection algorithm for 2-D seismic data. Let seismic data be represented by a plane harmonic wave propagating in an arbitrary direction;

$$f(t,x) = a.exp(-ik_x x - iwt) \qquad (1)$$

where $k_x$ is the horizontal wave number and $w$ temporal frequency, $x$ and $t$ are distance and time coordinates. Partial derivatives with respect to x and t will give;

$$\frac{\partial f(t,x)}{\partial x} = -ik_x f(t,x) \qquad (2)$$

and

$$\frac{\partial f(t,x)}{\partial t} = -iwf(t,x) \qquad (3)$$

the ratio of two derivatives will give;

$$\frac{\partial f(t,x)}{\partial x} / \frac{\partial f(t,x)}{\partial t} = \frac{k_x}{w} = \frac{\partial x}{\partial t} \qquad (4)$$

by Eikonal relationship, we have;

$$\frac{k_x}{w} = \frac{\partial t}{\partial x} = \frac{sin(q)}{V} \qquad (5)$$

where $q$ is the propagation angle measured with respect to the vertical and $V$ is the propagation velocity. Equation 5 shows the geometrical relationship of the recorded and propagating wave. The time gradient we observe on the data can be computed as the ratio of derivatives in x and t directions. Propagation direction is the where the most rapid change of phase and recorded amplitude takes place. The direction perpendicular to $q$ will have constant amplitude and phase, which means data continuity. This leads us to conclude that, by computation of derivatives of the seismic wave field can we determine the directions of most change ( discontinuity) and most similarity ( continuity ). The similarity is more conveniently measured over several adjacent traces ( velocity spectra measures the similarity of CDP trace gathers) by computing the semblance. Subsequently scanning for maximum semblance will provide the desired similarity and direction measurements. Edge detection on the other hand, is designed to detect sharp discontinuities of the wave field. This technique has been developed for satellite picture processing. In the noise free cases differentials are computed as differences between adjacent samples in orthogonal directions. Noisy cases require some filtering or weighted averaging of adjacent areas to minimize the noise effects. One of the weighting schemes used by the Delphi group is cubic polynomials. We use a Gaussian shaped weighting scheme with user controlled length. Let * represent convolution, and $W(t)$ represent the differentiating filter, then band limited differences in $x$ and $t$ directions are given by;

$$\Delta F_x(x,t) = W(t) * F(x-t,t) \quad \text{and}$$
$$\Delta F_t(x,t) = W(t) * F(x,t-t) \qquad (6)$$

where, $F(x,t)$ is the 2-D function for which we wish to determine the edges. Equations 6 represent the changes of the function only in $x$ and $t$ directions. The total change is;

$$C(x,t) = \sqrt{(\Delta F_x^2(x,t) + \Delta F_t^2(x,t))} \qquad (7)$$

and the direction of the maximum change is given by;

$$f(x,t) = arctan\{\Delta F_t(x,t), \Delta F_x(x,t)\} \qquad (8)$$

Once the function $C(x,t)$ is determined, all peaks of the function are determined. The magnitude of these peaks represent various degrees of discontinuity of the function $F(x,t)$.

These equations can easily be updated to 3-D functions as;

$$\Delta F_x(x,y,t) = W(t) * F(x-t, y, t) \quad ,$$
$$\Delta F_y(x,y,t) = W(t) * F(x, y-t, t) \quad \text{and}$$
$$\Delta F_t(x,y,t) = W(t) * F(x, y, t-t) \tag{9}$$

and the maximum change is given by;

$$C(x,t,t) = \sqrt{(\Delta F_x^2(x,y,t) + \Delta F_y^2(x,y,t) + \Delta F_t^2(x,y,t))} \tag{10}$$

The angle with respect to a horizontal plane of the normal to the discontinuity surface;

$$q(x,y,t) = arctan\{\Delta F_t(x,y,t), \sqrt{\Delta F_x^2(x,y,t) + \Delta F_y^2(x,y,t)}\} \tag{11}$$

The direction of maximum rate of change (azimuth);

$$f(x,y,t) = arctan\{\Delta F_y(x,y,t), \Delta F_x(x,y,t)\} \tag{12}$$

**MULTI-COMPONENT EDGE DETECTION:**

Equations 6 through 12 give the essence of the edge detection algorithm. In a multi-dimensional case the change of functions can be detected in the form of vectorial quantities. Consider a 3-D seismic data set, where each data sample can be represented by a number of attributes. These attributes could be physical, geometrical, pre-or-post stack attributes. As each sample can be defined by a number of different attributes we can select a set of attributes which is appropriate for the type of discontinuity we wish to determine.
In multi-component cases there are two practical difference detectors; vector dot product and Euclidean distance. Vector dot product, if the vectors are unitized, gives the cosine of the angle between the two vectors, a zero output means vectors in orthogonal directions, and a negative value indicates the opposite direction. Therefore the values should change between 1.0 and -1,0. We normalize the input attributes to zero mean and RMS=1 to minimize the possibility of one or more attribute dominating over the rest. Since the maximum occurs when two vectors are collinear, then vector dot product is better for continuity detection, in a similar manner to edge detection. This feature can be used in tracking events. For edge detection, we determine the function minima.
Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ represent two vectors given in N dimensional space,

$$X = [x_1, x_2, x_3, ......, x_N] \quad \text{and}$$
$$Y = [y_1, y_2, y_3, ......, y_N] \tag{13}$$

Then, the vector dot product is given by;

$$X.Y = \sum_{n=1}^{N} x_n . y_n \tag{14}$$

The second method, Euclidean distance computation, gives values opposite to the dot product. Zero distance indicates collinearity of two vectors. All distances larger than zero indicate various amounts of local discontinuity. For edge detection, we determine the location of local maxima The Euclidean distance is computed by;

$$\Delta\overline{X}\overline{Y} = \sqrt{\sum_{n=1}^{N}( x_n - y_n )^2} \quad (15)$$

The amount of maximum change, the angle to the maximum change and its azimuth are computed as given by the equations above.

In the presence of excessive noise, one dimensional filters can be applied in the direction of differentiation. This corresponds to weighted summation of vector components. Thus, a single vector is formed that goes into the computation.

**DISCUSSIONS:**

The multi-component edge detection algorithms discussed above can be directed to highlight discontinuities in a number of ways. It is understood that any algorithm detecting edges may also be used to show the lack of them, the spatial continuity. Vector dot product has its maximum value when two vectors are in phase. This corresponds to the highest similarity of two vectors. Therefore the dot product may be effectively used for event tracking. On the other hand, Euclidean distance computed between two vectors is minimized ( approaches zero distance ) when the two vectors are in phase. This distance increases as the vectors point in different directions. Therefore, Euclidean distance may be more useful for edge detection than the dot product.
Edges are detected by the vector differences in spatial and time directions. These derivatives will give us the quantitative value for continuity or discontinuity and its dip (and azimuth in 3-D). All of these additional measures can be incorporated into an attribute file that could indicate magnitude and direction of the discontinuity or continuity. Since the edges are the boundaries with largest change of features, they could represent boundaries with structural, stratigraphic and lithological implications.

**CONCLUSIONS:**

We have presented a generalized method of objective oriented multi-component edge detection. Initial results are presented in 2-D seismograms, which can easily be expanded to pre-stack and 3-D seismic data.
We have pointed out that each seismic data sample ( pre-stack, stacked or migrated) can be represented by an N dimensional vector. This definition can be used to detect discontinuities of various attribute combinations of a structural, stratigraphic or lithological nature. Several, optimally chosen attributes will detect discontinuities with lower degrees of uncertainty. Noise problems may be overcome by proper choice of average vector estimation in spatial and time direction.

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