A new travel time estimation method for horizontal strata
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Summary
As exploration targets get deeper, cable lengths increased accordingly. Increasing offsets have made the conventional two term hyperbolic equation produce increasingly erroneous travel times. As a remedy many researchers proposed various forms of hyperbolic equation and higher order of expansion. The fourth power expansions have been the most commonly used for NMO correction for the long offsets.

We have looked at the same problem, from a more Physical point of view. It is well known that the propagation velocity increases with offset, due to the waves spending longer travel paths in higher velocity layers. This increase can be expressed as the acceleration of the velocities with respect to offset. Hence we have incorporated this increase by adding an acceleration term to the velocity in the conventional two term travel time equation. We have tested this equation with a number of synthetic data and compared it to the results of two and three term time-distance equations.

We show in this paper that the results are at least as good as or better than the two or three term travel time equations. We also show that our equation explicitly expands to higher orders.

Introduction
Many papers have been published dealing with non-hyperbolic travel times. These became particularly noticeable when the industry started using very long offsets. Conventionally-corrected data usually shows an over correction, which displays itself in the familiar "hockey stick" effect. This over correction is of course, very detrimental to imaging and AVO processes. Many authors have looked at this effect from the physical side and gave equations based on various models of anisotropy (Tsvankin and Thomsen, 1994). Yet others gave solutions with higher order terms to fit the observed travel times. In many instances these did not correct all of the observed NMO and at the end some additional residual corrections had to be applied. Like many others, we have looked into 3-term NMO equations. We found that while the 3-term approach to NMO generally produced a smaller residual moveout than the basic 2-term equation, the applications could encounter practical limitations. There was also room for even improvement in accuracy. The alternative approach presented here offers ample potential in this respect.

It is well known that propagation velocities generally increase with offset. This increase generally gives rise to non-hyperbolic travel times. Because these velocities increase with offset, we attempt here to develop a simple expression based on a simple "velocity acceleration" term, rather than one directly based on an anisotropic model. Here we show that such a simple expression accurately predicts travel time curves in the case of horizontal stratification; the new formula is also easy to apply.

Previous Work
In a previous paper (Taner and Koehler, 1969) the time-offset relationship was given as a two-term Taylor series

\[ T^2(x) = T^2(0) + \frac{x^2}{V_{RMS}^2} \] (1)

Taner and Koehler (1969) and Al-Chalabi (1973) also gave the three-term expansion

\[ T^2(x) = T^2(0) + \frac{x^2}{V_{RMS}^2} + \frac{(1 + \frac{V_4^4}{V_{RMS}^4})x^4}{4T(0)V_{RMS}^2} \] (2)

where \( V_4 \) is the fourth-order velocity moment

\[ V_4 = \frac{1}{T(0)} \sum v^4(i) \Delta t(i) \] (3)

where \( v \) is interval velocity. Later Alkhalifah (1997) and others gave a similar three-term relationship,

\[ T^2(x) = T^2(0) + \frac{x^2}{V_{RMS}^2} + \frac{2\eta x^4}{V_{RMS}^2 + \sqrt{V_{RMS}^4 V_4^4 (0) + (1 + 2\eta)x^2}^2} \] (4)

where \( \eta \) is a travel-time dependent anisotropy coefficient.

All these approaches can be cast into the general form

\[ T^2(x) = A + Bx^2 + Cx^4 \] (5)

where \( A \) is the intercept time, or two way travel time, \( B \) is the inverse of the RMS squared velocity, and \( C \) can be read from the last term in either (2) or (4).
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Method

Let us now assume that the velocities increase with offset. If these increases are substantial, we expect large deviations from exact hyperbolic behavior. One way to handle this situation is to write \( V(x) \) as

\[
V(x) = V(0) + a x^2
\]  

(6)

where \( a \) is a parameter that describes the increase of velocity with offset, the "acceleration factor".

Then the total travel time is

\[
T^2(x) = T^2(0) + \frac{x^2}{\{V(0) + ax^2\}^2}
\]  

(7)

This expression is similar to the conventional hyperbolic equation (1), except for the acceleration term. In practice we must estimate \( V(0) \) and the \( a \) terms for each picked event.

To illustrate the new approach based on equation (8), we use a horizontally stratified medium. We computed travel times up to the occurrence of full refraction. For each case we computed the total travel time from source to receiver as well as the total offset and RMS velocity corresponding to this source-receiver offset. The second term in (8) allows us to write

\[
V(x) = \frac{x}{\sqrt{T^2(x) - T^2(0)}}
\]  

(8)

We computed \( V(x) \) velocities for each model for all relevant offsets. Then we least-squares fitted the velocities obtained from (1) to the velocity acceleration model by minimizing

\[
\epsilon^2 = \sum \{V(x) - V(0) - a x^2\}^2
\]  

(9)

and thereby obtained least mean squares (LMSQ) optimal values for both the parameters \( a \) and \( V(0) \). The resulting estimates were then inserted into (7), thus enabling us to study the errors produced by our new formulation.

Synthetic Examples

We have developed a synthetic model to test the accelerated velocity model given by (6). The maximum offset is around 18,000 m. The initial velocity is 1,500 m/s, with a velocity increment of 10 m/s per layer. Each layer is 20 m thick, and the model includes 400 layers in all. We computed 100 offsets; our maximum initiation angle is 0.995 of the angle that would produce a reflection from the deepest and hence highest velocity layer.

Figure 1 shows an example for a horizontally stratified medium. Figure 2 depicts the errors between the travel times obtained from the classical formulation given by Eq. (1) and from the new accelerated velocity formulation (7).

As we can see on Figure 1, the predicted and computed velocities to each receiver position are in close agreement. Figure 2 confirms this, where we note that the error between the actual travel times and the travel times predicted with Eq. (8) are quite small.

Concluding Remarks

We have carried out a large number of tests similar to the ones described above. They suggest that our simple formulation is at least as accurate as most previously published formulations. The next step is of course to apply the new approach (7) to real data sets. It is of some interest to note that it is possible to establish several approximate analytic relationships between the accelerated velocity model (7) and the original Taner and Koehler (1969) expansion

\[
T^2(x) = T^2(0) + c_2 x^2 + c_3 x^4 + c_4 x^6 + \ldots
\]  

(10)

One of these can be shown to be

\[
T^2(x) = T^2(0) + \frac{x^2}{V_o^2} - 2a \frac{x^4}{V_o^4} + b(x^6)
\]  

(11)

where \( a \) is the acceleration factor given in Eq. (7). We shall be exploring some instructive relationships of this kind in a paper currently in preparation. Our numerical experiments suggest that we can obtain considerable improvements over the conventional quadratic formulation of the travel time equation. No added computational complexities arise from our new model, even while we obtain considerably better fits during the course of extensive numerical experiments we have conducted thus far.

References

Al-Chalabi, M. 1973 , Series approximation in velocity and travel time computation; Geophysical Prospecting, 783-795.
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Figure 1: Travel velocity versus offset; $V(x)$ computed along the ray path; $V(x)$ computed for each receiver location, using Eq. (8); $V(x)$ computed to each receiver position after LMSQ fit using the accelerated velocity model given by Eq. (7).

Figure 2: Travel Time Errors; curve 1 is the travel time error between actual and predicted times according to Eqs. (1) and (7), respectively; curve 2 is the error in the hyperbolic function determined by fitting to one third of the total available offset; curve 3 is the error in the hyperbola fitted to two thirds of the total offset; curve 4 is the error in the hyperbola fitted to the whole data set, now for all offsets.