Yet another Vs equation

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Summary

The classical Raymer-Hunt-Gardner (1980) functional form $V_p = (1 - \phi)^2 V_{ps} + \phi V_{pf}$ (RHG), where $V_{ps}$ and $V_{pf}$ denote the $P$-wave velocity in the solid and in the pore-fluid phases, respectively, and $\phi$ is the total porosity, can also be used to relate the $S$-wave velocity in dry rock to porosity and mineralogy as $V_{s\text{dry}} = (1 - \phi)^2 V_{s\text{dry}}$, where $V_{s\text{dry}}$ is the $S$-wave velocity in the solid phase. Assuming that the shear modulus of rock does not depend on the pore fluid, $V_s$ in wet rock is $V_{sw} = V_{s\text{dry}}\sqrt{\rho_{\text{dry}}/\rho_{w}}$, where $\rho_{\text{dry}}$ and $\rho_{w}$ denote the bulk density of the dry and wet rock, respectively. This new functional form for $V_s$ prediction reiterates Nur’s (1998) critical porosity concept: the $V_s/V_p$ ratio in dry rock equals that in the solid phase. However, the velocity-porosity trend that follows from this equation somewhat differs from the traditional critical porosity trend.

Introduction: S-Wave Velocity Predictors

Historically, the input to $S$-wave velocity equations has been $V_p$ rather than porosity. Almost all such equations are empirical and derived for wet sediment. Picket (1963) showed that in limestone $V_s = V_p/1.9$ while in dolomite $V_s = V_p/1.8$. Later, Castagna et al. (1993) modified these relations: $V_s = 0.555V_p^2 + 1.017V_p - 1.031$ for limestone and $V_s = 0.583V_p - 0.678$ for dolomite, where the velocity is in km/s. In the same paper the equation for clastic rock reads $V_s = 0.804V_p - 0.856$. The Castagna et al. (1985) famous “mudrock line” gives $V_s = 0.862V_p - 1.172$.

Han (1986) used an extensive sandstone experimental dataset with large ranges of porosity and clay content variation to obtain $V_s = 0.794V_p - 0.787$. These measurements were conducted on wet rock at ultrasonic frequency.

Mavko et al. (1998) added to these measurements a number of data points from high-porosity unconsolidated sands and obtained $V_s = 0.79V_p - 0.79$. Further analysis of Han’s (1986) data yields $V_s = 0.754V_p - 0.657$ for rock where the clay content is below 0.25 and $V_s = 0.842V_p - 1.099$ where it exceeds 0.25.

If the same dataset is parted according to porosity ranges, it gives $V_s = 0.853V_p - 1.137$ for porosity below 0.15 and $V_s = 0.756V_p - 0.662$ for porosity exceeding 0.15.

Williams (1990) used well log data to arrive at $V_s = 0.846V_p - 1.088$ for water-bearing sands and $V_s = 0.784V_p - 0.893$ for shales.

Greenberg and Castagna (1992) combined relations for various lithologies to provide a unified empirical transform in multimineral brine-saturated rock composed of sandstone, limestone, dolomite, and shale. Their prediction can be also used for rock with any pore fluid if we assume that the shear modulus is not influenced by the pore fluid and apply Gassmann’s fluid substitution to the bulk modulus.

A different, theory-oriented, approach to $V_s$ prediction simply assumes that in dry rock, the ratio of the bulk to shear modulus is exactly the same as in the solid (mineral) phase. This automatically means that $V_{sw}/V_{ps} = V_{s\text{dry}}/V_{ps\text{dry}}$. This result approximately matches Picket’s (1963) data and was first utilized by Krief et al. (1990). Expressions for saturated rock can be simply obtained by combining this relation with Gassmann’s equations. Instead, Krief et al. (1990) suggested $(V_{ps\text{sat}} - V_{pf})/V_{ps\text{sat}} = (V_{s\text{sat}} - V_{f})/V_{s\text{sat}}$, where $V_{ps\text{sat}}$ and $V_{s\text{sat}}$ are the $P$- and $S$-wave velocity in saturated rock, respectively; $V_{ps}$ and $V_{s\text{dry}}$ are those in the solid phase; and $V_{pf}$ is the velocity in the pore fluid.

For completeness, we need to mention the Xu and White (1995) model for velocity prediction in shaley sediment that follows an intricate scheme of assuming that the quartz and clay components of rock are materials with ellipsoidal pores of prescribed aspect ratios, calculating the elastic moduli of these components using an effective-medium theory, and then calculating the elastic moduli of the rock by a differential-effective-medium scheme. Later, Keys and Xu (2002) published an “approximation for the Xu-White model” using the same approach and giving it a comprehensive mathematical treatment.

Another “simple” $S$-wave velocity predictor is due to Lee (2006). It amounts to relating the dry-rock shear modulus to its bulk modulus via a “consolidation constant” which itself has to be somehow pre-assigned.

Raymer-Hunt-Gardner for S-Wave velocity

The original RHG equation only provides for the $P$-wave...
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Yet another V<sub>s</sub> velocity calculation. Here we add an ad-hoc RHG equation for the S-wave velocity by assuming that in dry sediment \( V_{d,\text{dry}} = (1 - \phi)^2 V_{d,\text{s}} \), where \( V_{d,\text{s}} \) is, once again, the S-wave velocity in the solid phase.

The S-wave velocity in the wet sediment is then obtained from \( V_{d,\text{dry}} \) by assuming that the rock’s shear modulus is not affected by pore fluid (Gassmann, 1951):

\[
V_s = V_{d,\text{dry}} \sqrt{\frac{\rho_{\text{dry}}}{\rho_s}} = (1 - \phi)^2 V_{d,\text{s}} \sqrt{\frac{(1 - \phi)\rho_s + \phi \rho_f}{(1 - \phi)\rho_s + \phi \rho_f}},
\]

(1)

where \( \rho_{\text{dry}} \) and \( \rho_s \) are the bulk densities of dry and wet sediment, respectively; and \( \rho_f \) and \( \rho_s \) are the densities of the solid and fluid phase, respectively.

\( V_{d,\text{s}} \) (as well as \( V_{p,\text{dry}} \)) can be calculated from the elastic moduli and density of the appropriate mineral mix as the square root of the modulus divided by the density. \( \rho_s \) is the volume-weighted arithmetic average of the densities of the components while the elastic moduli of the mix are given by Hill’s (1952) average.

To check the quality of this prediction consider a subset of Han’s (1986) velocity data collected on room-dry mature clastic samples at 30 MPa differential pressure. We use Gassmann’s equations to calculate the velocity in these samples saturated with brine and gas with the fluid properties listed in Table 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>Bulk</th>
<th>Shear</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>36.600</td>
<td>45.000</td>
<td>2.650</td>
</tr>
<tr>
<td>Clay</td>
<td>21.000</td>
<td>7.000</td>
<td>2.580</td>
</tr>
<tr>
<td>Calcite</td>
<td>76.800</td>
<td>32.000</td>
<td>2.710</td>
</tr>
<tr>
<td>Dolomite</td>
<td>94.900</td>
<td>45.000</td>
<td>2.870</td>
</tr>
<tr>
<td>Brine</td>
<td>2.330</td>
<td>0.000</td>
<td>1.029</td>
</tr>
<tr>
<td>Gas</td>
<td>0.017</td>
<td>0.000</td>
<td>0.112</td>
</tr>
</tbody>
</table>

Table 1. Elastic moduli (in GPa) and density (in g/cc) of rock and fluid components. The properties of quartz, clay, calcite, and dolomite are from Mavko et al. (1998).

We also calculate \( V_p \) from RHG and employ it to predict \( V_s \) according to Greenberg and Castagna (1992) and Krief et al. (1990) (arguably the two most widely-used predictors). Finally, we calculate \( V_s \) from porosity and clay content according to Equation (1). These model curves are produced for quartz/clay sediment with the clay content zero, 0.5, and 1, saturated with the same brine or gas, and are superimposed on the data in Figure 1. The elastic constants and density of the minerals used in this modeling are listed in Table (1).

Figure 1: Han’s (1986) data color-coded by the clay content. Velocity versus porosity at 30 MPa for brine- (top) and gas-saturated (bottom) rock. The model S-wave velocity curves are according to (clockwise) Greenberg and Castagna (1992), Krief et al. (1990), and Equation (1), labeled “GC,” “Krief,” and “Raymer,” respectively. The upper curves are for pure quartz, the middle curves are for 0.5 clay content, and the bottom curves are for pure clay.

Notice that RHG accurately models \( V_p \). All \( V_s \) predictors used here, including Equation (1), produce accurate estimates as well. RHG slightly underestimates \( V_p \) in a
few pure-quartz samples and so do the \( V_s \) predictors. The predictions are off-mark for one high-porosity sample which is unconsolidated and almost pure quartz Ottawa sand (the dark-blue symbol of porosity 0.33). Nevertheless, we display this data point to further emphasize that neither the original RHG nor Equation (1) are suitable for soft unconsolidated sediment.

An important feature of Han’s dataset is that the reported clay content is the fractional volume of porous clay in the unit rock volume (\( c \)) rather than that of mineral clay in the unit solid-phase volume (\( C \)). The relation between the two is \( C = c(1 - \phi_{clay})/(1 - \phi) \), where \( \phi_{clay} \) is the intrinsic porosity of clay. Figure 2 shows how \( C \) relates to \( c \) for varying \( \phi_{clay} \) and \( \phi \). Clearly, for a large clay content one may deviate from the other which needs to be taken into account when comparing these data and model curves because the latter use \( C \) rather than \( c \).

Figure 2: The volume fraction of mineral clay in the solid phase versus that of porous clay in the unit volume of rock for the assumed intrinsic porosity of clay zero, 0.1, 0.2, and 0.3 (clockwise). The four lines in each frame are for the total porosity of the rock 0.3, 0.2, 0.1, and zero (from top to bottom).

Consider next data by Rafavich et al. (1984) which include the velocity, porosity, and mineralogy of about 30 room-dry carbonate samples measured at 30 MPa differential pressure. These low-porosity (between zero and 0.1) samples contain between 0.7 and 1.0 calcite with the rest being predominantly dolomite.

After fluid substitution for brine and gas, these data are displayed in Figure 3 together with the pure-calcite model curves. RHG slightly overestimates the compressional-velocity data, however, the Equation (1) curves accurately model \( V_s \). So does the Krief et al. (1990) predictor. However, the Greenberg and Castagna (1992) predictor slightly misses the zero-porosity end-point for calcite.

Figure 3: The Rafavich et al. (1984) data color-coded by the calcite content. The open red symbols are for chalk (Brevik, 1995). Velocity versus porosity at 30 MPa for brine- (top) and gas-saturated (bottom) rock. Left column is for the \( P \)-wave velocity while the right column is for the \( S \)-wave velocity. The model \( S \)-wave velocity curves are according to Greenberg and Castagna (1992), Krief et al. (1990), and Equation (1), labeled “GC,” “K,” and “R,” respectively. The heavy curves in the right column are from Equation (1). All model curves are for pure calcite.

The open red symbols in Figure 3 are for chalk, courtesy of Brevik (1995). Although, these data are at low differential pressure, we nevertheless display them for comparison. RHG overestimates the \( P \)-wave velocity in chalk which once again underscores the fact that it is inappropriate for high-porosity unconsolidated sediment.

However, all three \( V_s \) predictors provide fairly close estimates for the chalk data. This is somewhat peculiar for the two older methods (a correct answer from incorrect input) but encouraging for the users of Equation (1) because it predicts \( V_s \) (correctly in this case) not from \( V_p \) but from porosity, lithology, and fluid.

Finally, let us examine a dolomite-rich subset of the Kenter et al. (1997) ultrasonic velocity data obtained on wet outcrop samples with dolomite, calcite, quartz, and clay at
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30 MPa differential pressure. In Figure 4 these data are compared to the model curves produced for wet conditions.

Once again, RGH, as well as the three \( V_s \) predictors, fairly accurately match the data, perhaps except the Greenberg and Castagna (1992) predictor which misses the zero-porosity end-point for dolomite. Once again, a positive feature of the Equation (1) method is that it predicts \( V_s \) directly from the rock’s bulk properties rather than from \( V_p \).

Discussion and Conclusion

The new \( V_s \) equation introduced here is nothing more than an ad-hoc extension of the original RHG functional form that links \( V_s \) to porosity, mineralogy, and fluid. Yet, it mimics selected data with reasonable fidelity and calculates \( V_s \) directly from porosity, mineralogy, and fluid rather than from \( V_p \). Therefore, its use is justified in consolidated-rock environments once its applicability is verified by high-quality site-specific data.

We recommend that first-principle-based rock physics models be always used to calibrate and understand the velocity-porosity behavior, or, in other words, “diagnose” the rock, simply because these models allow for reasonable selection of such parameters as pressure, contact stiffness, and the number of contacts per grain. However, this recommendation does not preclude the use of empirical and/or ad-hoc simple expressions but only if they match selected data and, in addition, can be justified by more involved theoretical models.

Equation (1) meets these criteria. It accurately matches data from mature rock and is justified by the theoretical “stiff-sand” (Gal et al., 1999) model (Figure 5). Therefore, it can be included into the arsenal of rock-physics tools.

This equation, if considered in combination with the original RHG, reiterates the critical porosity concept (Nur et al., 1998) that the \( V_p/V_s \) ratio in dry porous rock is the same as in its solid phase. However, it produces a different velocity-versus-porosity curve (Figure 5).

Finally, this new S-wave predictor is sensitive to the mineral elastic constants and density. Those listed in Table 1 are commonly employed in rock physics analyses but do not preclude the user from selecting a reasonable alternative. Still, we strongly recommend caution when using the elastic moduli and density of quartz, clay, calcite, and dolomite as free variables for matching, e.g., well sonic and dipole data with model-derived curves.

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REFERENCES


