An anisotropic model for the electrical resistivity of two-phase geologic materials

Michelle H. Ellis\(^3\), Martin C. Sinha\(^2\), Tim A. Minshull\(^2\), Jeremy Sothcott\(^1\), and Angus I. Best\(^1\)

**ABSTRACT**

Electrical and electromagnetic surveys of the seafloor provide valuable information about the macro and microscopic properties of subseafloor sediments. Sediment resistivity is highly variable and governed by a wide range of properties including pore-fluid salinity, pore-fluid saturation, porosity, pore geometry, and temperature. A new anisotropic, two-phase, effective medium model describes the electrical resistivity of porous rocks and sediments. The only input parameters required are the resistivities of the solid and fluid components, their volume fractions and grain shape. The approach makes use of the increase in path length taken by an electrical current through an idealized granular medium comprising of aligned ellipsoidal grains. The model permits both solid and fluid phases to have a finite conductivity (useful for dealing with surface charge conduction effects associated with clay minerals) and gives results independent of grain size (hence, valid for a wide range of sediment types). Furthermore, the model can be used to investigate the effects of grain aspect ratio and alignment on electrical resistivity anisotropy. Good agreement was found between the model predictions and laboratory measurements of resistivity and porosity on artificial sediments with known physical properties.

**INTRODUCTION**

Measurements of electrical resistivity are commonly used in petroleum exploration to investigate the nature of sedimentary formations and especially the water saturation in hydrocarbon reservoirs. Geophysical techniques that use electrical resistivity include borehole induction logging, a well-established wireline logging method, and seafloor controlled-source electromagnetic (CSEM) surveying, which is emerging as a powerful hydrocarbon exploration tool. The interpretation of such resistivity measurements relies on knowledge of the relationships between the effective (bulk) resistivity and the physical properties of the constituent parts of the sediment (i.e., solid minerals and interstitial fluid). The electrical resistivity of a sediment is primarily controlled by porosity, pore fluid saturation, pore fluid salinity, temperature, mineralogy (quartz, feldspar, clay, carbonate, etc.), and grain fabric (e.g., grain shape and alignment).

The standard method of interpreting electrical resistivity data for sedimentary sequences, especially in well log interpretation, involves the use of Archie’s (1942) equation. Archie (1942) showed experimentally that the resistivity of clean sandstone is proportional to the resistivity of the brine saturating the sandstone. The proportionality constant is known as the formation factor \( F \) and can be related to the porosity of saturated sandstone using the empirical relationship

\[
F = \frac{\rho_a}{\rho_f} = a\varphi^{-m},
\]

where \( \rho_f \) and \( \rho_a \) are the resistivities of the fluid and the fully saturated sedimentary medium respectively and \( m \) and \( a \) are empirical constants. The constant \( a \) represents the “tortuosity factor” of the system, \( m \) is the “cementation exponent,” and \( \varphi \) is the porosity. The advantage of this approach is its simplicity. The empirical constants within the equation can be varied such that it can fit almost any data set. Formation factor versus porosity curves can be determined for different lithologies, and these are used extensively in the hydrocarbon industry (Schlumberger, 1977). Archie’s equation is however purely empirical, and so the cementation and tortuosity constants cannot be physically justified. In addition, a large data set is usually needed to determine these constants, although approximate values have been determined for several different types of sediments (Schlumberger, 1977).

Using a physical model to estimate resistivity is more desirable than empirical techniques for several reasons. First, empirical tech-
niques typically involve constants and must be derived from an ini-
tial data set. The initial data set may not span the entire porosity
range, and the empirical constants calculated from the initial data
cannot always be used outside the original data range (Berg, 1995),
thus leading to errors. Second, the empirical constants determined
for one data set cannot necessarily be applied to another data set.
Third, because the constants can be freely varied, they can be forced
to fit almost any data set, which may lead to incorrect interpretation.
Using a physical model is also desirable because the model can be
made physically compatible with one or more of the seismic effec-
tive medium models in widespread use. Physically consistent elec-
trical and seismic effective medium models would be useful for
jointly interpreting combined geophysical data sets and would lead
to a more complete understanding of the nature of the sediment in-
vestigated.

There are very few effective medium models for resistivity that
are purely physical and contain no empirically derived constants that
describe how the resistivity of the composite is computed from the
resistivity of the components. To predict the effective resistivity of
a multicomponent medium, three requirements must be specified
(Mavko et al., 2003): (1) the volume fraction of each of the compo-
ients; (2) the electrical resistivity of each of the components; and (3)
the geometrical relationship between the components. The most
widely used purely physical models are the Hashin-Shtrikman (HS)
conductivity bounds (Hashin and Shtrikman, 1962) and the Hanai-
Bruggeman (HB) equation (Hanai, 1960; Bruggeman, 1935). One of
the main advantages these models have over Archie’s equation is
that they allow the solid phase to have a finite conductivity. This is
particularly important when estimating the electrical resistivity of
sediments containing clay minerals. Clay minerals can conduct
charge through electric double layers and have large surface area to
volume ratios, causing them to contribute significantly to the effec-
tive conductivity (reciprocal of resistivity) of the sediment. Resistiv-
ity models for clays developed by Wyllie and Southwick (1954),
Waxman and Smits (1968), Clavier et al. (1984), Bussian (1983),
and Berg (1995) all suffer from the same limitation as Archie’s equa-
tion, namely they all require at least one empirically derived con-
stant to account for the grain’s geometrical arrangement. Several of
the models require multiple empirical constants.

One of the main limitations of the HS method is that the geometri-
cal relationship between the matrix and the pore fluids is not taken
into account; as a result, only upper and lower bounds of resistivity
are determined. In addition, because the geometry between the
different phases is not specified, the models must assume the effec-
tive medium is isotropic. The HB equation uses a differential effec-
tive medium (DEM) approach to determine the effective resistivity
of a medium that contains spherical inclusions. Unlike the HS
bounds, the HB equation specifies a geometrical relationship between
the phases; however, several problems exist with the HB bound.
First, it assumes the solid phase is totally interconnected while the
fluid phase inclusions are isolated, which is unrealistic in nature;
however, Sen et al. (1981) used a similar DEM method that reverses
the solid and fluid phases to overcome this problem. Second, only
spherical inclusions are used within the model, which again is not al-
ways realistic when representing a sediment. Third, as a conse-
quence of the spherical inclusions, the model requires that the final
effective medium is isotropic. In real geological formations, aniso-
tropy can be caused by the alignment of sediment grains with aspect
ratios less than one. This alignment often occurs in shales and mud-
stones, leading to large degrees of electrical anisotropy (Anderson
and Helbig, 1994; Clavaud 2008). It is therefore important to devel-

In this paper, we present an electrical effective medium model for
uncemented fully saturated sediments which is based on the physical
arrangement of the phases, in which the conducting fluid is intercon-
nected, and in which we specify the shape and alignment of the
grains. The final effective medium represents an idealized pack of el-
lipsoids, representing the sediment grains, in a conductive medium.
The physical parameterization of the medium is identical to that used
in a number of widely applied seismic effective medium models. In
the second part of the paper, we validate our model by comparing its
predictions to laboratory measurements made on artificial sedi-
ments.

**ELECTRICAL EFFECTIVE MEDIUM MODEL**

The presence of relatively resistive grains in a conductive fluid in-
fluences the effective resistivity in several ways:

1) The grains reduce the cross-sectional area of conduction
through which the electric current must flow. This also means
that the current density through the more conductive phase is
increased while the current density through the proportion of
the original volume now occupied by the more resistive phase
is decreased.

2) Since in general the current is no longer directly aligned
with the ambient electric field, there is an increase in the “path-
length” as the current will preferentially travel around the
grains rather than through them.

3) The number density of particles influences the proportion of
the path length which is deviated in order to travel around the
grains and the proportion of the path length which is not
deviated.

This section develops an electrical effective medium model that
takes into account all three of these factors but remains purely physi-
cally based (i.e., requiring no empirical constants).

**Hashin-Shtrikman bounds**

The first factor can be accounted for by using the Hashin-Shtrik-
man (HS) resistivity bounds (Hashin and Shtrikman, 1962) as the
starting point for our model. The HS bounds give the narrowest pos-
sible isotropic bounds without defining the geometry between com-
ponents of a two-phase medium. All the components within this me-
dium are isotropic and homogeneous. The upper bound represents
the maximum conductivity the isotropic composite can have. This
occurs when the fluid (conductive phase) is totally interconnected
and the solid (resistive phase) occurs as totally isolated inclusions.
The HS lower or resistive bound represents the bound when the fluid
phase occurs as completely isolated inclusions and the solid phase is
totally interconnected. No other information is given about the geo-
metry system. The HS bounds are given by

\[ \frac{1}{\rho_{\text{HS,conductive}}} = \sigma_{\text{HS,conductive}} + \sigma_f (1 - \beta) \left( \frac{1}{\sigma_s - \sigma_f} + \frac{\beta}{3\sigma_f} \right)^{-1}, \]

\[ \sigma_s = \sigma_f + (1 - \beta) \left( \frac{1}{\sigma_s - \sigma_f} + \frac{\beta}{3\sigma_f} \right)^{-1}, \]
\[
\frac{1}{\rho_{HS,\text{resistive}}} = \sigma_{HS,\text{resistive}} = \sigma_z + \beta \left( \frac{1}{\sigma_f - \sigma_z} + \frac{1 - \beta}{3\sigma_z} \right)^{-1},
\]
where \(\sigma_{HS,\text{conductive}}\) is the upper or conductive HS bound of effective conductivity, \(\sigma_{HS,\text{resistive}}\) is the lower or resistive HS bound of effective conductivity, and \(\rho_{HS,\text{conductive}}\) and \(\rho_{HS,\text{resistive}}\) are the conductive and resistive HS bounds of the effective resistivity respectively; \(\sigma_z\) and \(\sigma_f\) are the conductivities of the solid and fluid respectively and \(\beta\) is the volumetric fraction of the fluid (assumed to be equal to the porosity). The physical relationship between the pore fluids and the solid grains in an uncemented clastic sediment is represented best by the HS conductive bound (rather than the resistive bound) where all the fluid is interconnected; therefore, we use the conductive bound as the starting point for our model.

Geometric factor

The second factor can be addressed by investigating the influence of the average increase in path length that the electric current has to take to pass through the sediment. This can be represented as a geometric factor. Herrick and Kennedy (1994) used the path a current takes through a formation to determine the effective resistivity. Their model assumes that the formation can be represented as a solid volume (representing the matrix) with a series of tubes running through it representing the pores. A geometrical parameter can be calculated from the size, shape, and number of tubes and then used to determine the effective resistivity of the formation. The problem with this method is that it cannot represent completely the complex pore geometry observed in sediments. Rather than trying to model the complex shapes of pores, the geometric factor developed in this paper concentrates on estimating the change in current path caused by the grains. In practice, the electric current will typically take the form of mobile ionic charge carriers (anions and cations) dissolved in the pore fluid. The charge carriers, and therefore electric current, will take the shortest available route through the sediment along the direction of the imposed electric field; but this is longer than the actual length of sediment because the current must go around the grains (Figure 1).

To estimate the increase in path length, the grains of a sediment are idealized. A sphere — an ellipsoid with an aspect ratio of one — is the simplest shape and can be used to model a sand grain. Taking a simple example, suppose a charge carrier encounters a spherical grain at its center. The charge carrier will be deviated at most around half of the circumference of the grain and leave it at its opposite “pole” (Figure 1). If the diameter of the sphere is \(d\), then the undeviated path length \(\ell_1\) would also be \(d\); however, the deviated path length \(\ell_2\) is half the circumference, \(\pi d/2\). The geometric factor, \(\ell_1/\ell_2\), is then given by \(\pi d/2d = \pi/2\) or approximately 1.57. Assuming therefore that the increase in path length is due to a spherically shaped grain, the increase can be up to 57%, causing the resistivity of the medium to increase by the same amount.

This value of \(\pi/2\) assumes that the electric current encounters the sphere at its center and then travels around the grain to the opposite pole, at which point the current continues along its original path. However, the current (charge carrier) may encounter the grain at any point on the grain surface which is in the path of the current; therefore, the average increase in path length for randomly distributed charge carriers needs to be calculated. It is assumed that when the current encounters the grain it will travel around the grain until it reaches a point at which it can continue in the fluid along its original path. This requirement forces the current to redistribute itself as it passes around the grain, and imposes a condition on the model that, once clear of the grain, the electric current density within the fluid phase becomes uniform and aligned along the direction of the imposed electric field, until another grain is encountered. Using this simple model, we can assume that (1) a current path that does not directly encounter a grain will suffer no deviation and (2) a current path that encounters a grain is deviated around its circumference until it reaches the corresponding point on the other side.

To calculate the geometric factor, that is, the fractional increase in path length, the deviated path length \((\ell_1)\) and the undeviated path length \((\ell_2)\) need to be calculated (Figure 2).

In this case,
\[
\ell_1 = 2\left[\frac{\pi r}{2} - r \gamma \right] = 2\left[\frac{\pi r}{2} - r \sin^{-1}\left(\frac{w}{r}\right)\right],
\]
\[
\ell_2 = 2(r^2 - w^2)^{1/2},
\]
where
\[
w = r \sin \gamma,
\]
\(r\) is the radius of the grain, \(w\) is the distance in the horizontal plane between the center of the grain and the point at which the current encounters the surface of the grain (Figure 2a and b) and \(\gamma\) is the angle at the center of the grain between the vertical and the point at which the current encounters the surface of the grain (Figure 2a).

The geometrical factor \(g\) can then be calculated as follows:
\[
g = \ell_1/\ell_2 = \frac{\frac{\pi r}{2} - r \gamma}{\sqrt{r^2 - (r \sin \gamma)^2}} = \frac{\frac{\pi}{2} - \gamma}{\sqrt{\cos^2(\gamma)}}.
\]
Because both \(\ell_1\) and \(\ell_2\) are proportional to the radius, the \(r\)’s cancel. The geometric factor is therefore independent of the size of the grain. This is an important result for the practical application of any effective medium model.

When viewed from above, the grain appears as a circle on which the current may hit at any point (Figure 2). To obtain the average geometric factor \(G\), the path length must be calculated at every point over the grain’s cross section:

![Figure 1. The deviation of electric current around a spherical grain.](image-url)
merically, we determine that for a sphere the geometric factor is the grain center. The parameters point where the current encounters the grain, with the origin taken at tions 4–9. Evaluating this double integral numerically, we determine that for a sphere the geometric factor is 1.178 (4 s.f.).

\[
G = \frac{\ell_{ave}}{\ell_{ave}^2} = \int_{x=0}^{r} \int_{y=0}^{\sqrt{r^2-x^2}} \ell_1 \, dy \, dx
\]
\[
\int_{x=0}^{r} \int_{y=0}^{\sqrt{r^2-x^2}} \ell_2 \, dy \, dx
\]

\[w = (x^2 + y^2)^{1/2}.\]  

The parameters \(x\) and \(y\) are the corresponding coordinates of the point where the current encounters the grain, with the origin taken at the grain center. The parameters \(\ell_{ave}\) and \(\ell_{ave}^2\) are the average \(\ell\) and \(\ell^2\) values over the entire sphere. Evaluating this double integral numerically, we determine that for a sphere the geometric factor is 1.178 (4 s.f.).

Figure 2. Calculation of the path length (geometric factor in equations 4–9) for electric current traveling in the z-direction when encountering a spherical grain: (a) in a plane through the z-axis and (b) in the x-y plane.

The resistivity of the fluid is multiplied by the geometric factor to give a new, higher, fluid effective resistivity that accounts for the extra distance traveled by the electrical current. This new fluid effective resistivity is then used with the HS conductive bound to give a new geometric effective resistivity of the medium (\(\rho_{geo}\)):

\[
\frac{1}{\rho_{geo}} = \frac{\sigma_{f}}{G} + (1 - \beta) \left( \frac{1}{\sigma_s - \frac{\sigma_f}{G}} + \frac{G\beta}{3\sigma_f} \right)^{-1}
\]

Mean path length

The geometric factor, as calculated above, cannot simply be applied to the HS conductive bound at all porosities because this will always have the effect of increasing the estimated resistivity. This would cause the estimated resistivity of the medium to be greater than the resistivity of the fluid at 100% porosity. Therefore, a method is needed to determine the percentage of the fluid to which the geometric factor must be applied, so that at 100% porosity the geometric factor is not applied and thus address point 3.

The current will spend a certain proportion of the total path length being deviated around the grains, with the remainder of the path length being undeviated (as it passes through the pores). The individual proportions will depend on the porosity of the sediment.

To calculate the average distance traveled by the current between the grains, an adapted version of the mean free path — used in the kinetic theory of gases to calculate the average distance between molecule collisions — can be used. The definition of the mean free path \(L\) is taken as the length \(\ell\) of a path divided by the number of collisions in that path and is given as

\[
L = \frac{1}{\sqrt{2n_vS_c}}
\]

where \(n_v\) is the number density of particles and \(S_c\) is the effective collision cross section. The \(\sqrt{2}\) term is included because in kinetics the molecules are considered to be moving. In our application, all of the grains are stationary and therefore this term can be left out. In kinetics, \(S_c\) is given by \(\pi d^2\) where \(d\) is the diameter of the molecules. It is assumed that both the molecules involved in the collision have volume. In the case of an electric current encountering a grain, the electric current element (charge carrier) can be said to have an infinitely small cross section relative to the sediment grain, and the collision cross section will therefore be solely dependent on the cross section of the grain. Therefore, \(S_c\) will be given as \(\pi r^2\). The number density \((n_v)\) is given by

\[
n_v = \frac{n_g}{\pi r^2 \ell}
\]

where \(n_g\) is the number of grains and \(\pi r^2 \ell\) is the volume that the grains occupy \((r\) is the radius of the grains). The mean free path \(L\) is then given by

\[
L = \frac{1}{\pi r^2 n_v} = \frac{\ell}{n_g}\]
and for sediment with porosity $\beta$, the number of grains ($n_g$) is given by

$$n_g = \frac{\pi r^2 (1 - \beta)}{4/3 \pi r^3} = \frac{(1 - \beta) 4/3}{r},$$  \hspace{1cm} (14)$$

where $\pi r^2 (1 - \beta)$ is the volume of all the grains and $4/3 \pi r^3$ is the volume of a single grain. Therefore, for an imaginary electric current line passing through sediment with porosity $\beta$,

$$L = \frac{\ell}{\ell (1 - \beta) \sqrt{\frac{4}{3} r}} = \frac{4r}{3(1 - \beta)}. \hspace{1cm} (15)$$

In this equation, $L$ — the mean free path length — represents the mean distance between grain centers. $L$ can then be used to determine the deviated and undeviated proportions of the total path length through a composite medium. If we consider vertical current flow and grains of finite size, then the geometric relationship between the average undeviated path length $\ell_{2ave}$, grain radius $r$, and the mean free path $L$ can be seen in Figure 3.

The mean free path length decreases with decreasing porosity whereas $\ell_{2ave}$ remains the same regardless of porosity. As mean free path length decreases, the tortuosity of the current path increases because a greater proportion of the path is spent deviated around the grains. Using equations 15 and 8 for $L$ and $\ell_{2ave}$, we can calculate the proportion of the total path length deviated by grains and the proportion that passes straight through the fluid ($F_{\text{grain}}$ and $F_{\text{fluid}}$, respectively):

$$F_{\text{grain}} = \frac{\ell_{2ave}}{L}, \hspace{1cm} (16)$$

$$F_{\text{fluid}} = 1 - F_{\text{grain}}. \hspace{1cm} (17)$$

Because both $L$ and $\ell_{2ave}$ are proportional to the radius of the grains, the deviated and undeviated proportions are again independent of the grain radius and only dependent on the porosity. The resulting proportions of deviated and undeviated path length can now be used to navigate between the HS conductive bound (equation 2) and the geometrically altered HS conductive bound ($\rho_{\text{GPL}}$, equation 10):

$$\frac{1}{\rho_{\text{GPL}}} = \frac{1}{\rho_{\text{HS,conductive}}} F_{\text{fluid}} + \frac{1}{\rho_{\text{geo}}} F_{\text{grain}} \hspace{1cm} (18)$$

where $\rho_{\text{GPL}}$ is the geometric path length effective resistivity (Figure 4). It can be seen that at 100% porosity, $\rho_{\text{GPL}}$ and $\rho_{\text{HS,conductive}}$ are the same. As the porosity decreases, $\rho_{\text{GPL}}$ leaves the HS bound and moves toward $\rho_{\text{geo}}$.

**Changing grain aspect ratio to simulate electrical anisotropy**

Electrical anisotropy can be achieved using our geometric method by altering the aspect ratio of the grains. Instead of dealing with circles and spheres, we are now dealing with ellipses and ellipsoids; however, the problem remains tractable. An ellipsoid can be defined in Cartesian coordinates:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \hspace{1cm} (19)$$

The equation for an ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \hspace{1cm} (20)$$

where $a$, $b$, and $c$ are the semi-axes of the ellipse and ellipsoid. The semi-axes may have any length; however, in the following equations we shall consider the case where at least two of the axes have the same length.

Before the arc length can be determined, the shape of the grains needs to be defined. In this study, we investigate two grain shapes: oblate and prolate. Once the shape is defined, the orientation of the grains must be specified. In the following cases, the grains will be oriented so that the electric current is traveling parallel to the $c$-axis.
(Figure 5). The arc length $S$ for the ellipsoid can be given as

$$S = \int_{P}^{Q} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt,$$

where $P$ and $Q$ are the endpoints of the arc and $t$ is a parametric value. In the case of a current element (charge carrier) encountering the grain, $P$ and $Q$ are the points where the current starts and ceases to be deviated. As in the case of a sphere, the mean path length must be determined by averaging the path length over the whole surface of the grain. Equation 18, which was used in the case of the spherical grain, can be used again to determine $G$ for an ellipsoidal grain. In this case,

$$\ell_1 = S,$$

and

$$\ell_2 = 2z,$$

where $z$ is given by equation 20.

Again, we have integrated numerically the resulting expressions. For grains aligned in the least resistive direction (i.e., the current is traveling parallel to the long axis, Figure 5), relatively little variation exists between the prolate and the oblate grains (Figure 6). The largest change in the geometric factor is associated with oblate grains where the short axis is aligned in the direction of the current. This

grain alignment will cause the resistivity of the medium to increase dramatically as the aspect ratio of the grains decreases.

Once the average geometric factor $G$ has been determined, the mean free path length $L$ again needs to be calculated. This is slightly different to the mean free path of a sphere because the axes of the ellipsoid have different lengths. The cross-sectional area ($A_c$) and the volume ($V_e$) of the ellipsoid are given by

$$A_c = ab \pi,$$

and

$$V_e = 4/3 \pi abc.$$

Equations 21–25 assume the $c$-axis orientation is parallel to the direction of current flow. These equations can be worked through as was done in the case of the sphere (equations 11–15) to give the mean free path of ellipsoidal grains:

$$L = \frac{4c}{6(1-\beta)}.$$

The resistivity of the effective medium can then be determined in the same way as for spherical grains (equations 16–18). It can be seen that a marked change exists in the effective resistivity between the HS conductive bound and the geometric path length effective resistivity (Figure 7). The biggest change is seen when the oblate grains are oriented with their short axes aligned with the current. In this case, the resistivity increases dramatically as porosity and aspect ratio decrease.

The above method calculates the maximum and minimum resistivity of an anisotropic medium. In order to calculate the resistivity when the current direction is neither parallel nor perpendicular to the long axis of the grains, we use an adapted version of Price’s (1972) method. Originally used to determine the resistivity in any direction of an arbitrary-shaped sample of an anisotropic material, Price’s method was an extension of the van der Pauw (1958) method of measuring the resistivity of isotropic materials. For simplicity, we will

Figure 5. Relative lengths of the semi-axes of oblate and prolate grains when the grains are in their most and least resistive orientations. Current direction is always parallel to the $c$ semi-axis.

Figure 6. Geometric factor as a function of grain aspect ratio for fully aligned oblate and prolate grains in the most and least resistive alignments.
look at the two-dimensional problem in the x-y plane. In the isotropic case, when $\rho_x = \rho_y = \rho$, current density ($\mathbf{J}$) is given by

$$\mathbf{J} = \frac{\mathbf{E}}{\rho},$$

(27)

where $\mathbf{E}$ is the electric field and is parallel to $\mathbf{J}$ and orthogonal to the electric equipotentials. However, in the anisotropic case when $\rho_x \neq \rho_y$, $\mathbf{J}$ is now given as

$$\mathbf{J} = \frac{E_x \hat{x}}{\rho_x} + \frac{E_y \hat{y}}{\rho_y},$$

(28)

where $E_x$ and $E_y$ are components of the electric field in the x and y directions and $\hat{x}$ and $\hat{y}$ are unit vectors along the x- and y-axis, respectively. The amplitude of $\mathbf{J}$ is then given by

$$|\mathbf{J}| = \left[ \frac{E_x^2}{\rho_x^2} + \frac{E_y^2}{\rho_y^2} \right]^{\frac{1}{2}} = \left[ \frac{E_x^2 \rho_y^2 + E_y^2 \rho_x^2}{\rho_x \rho_y} \right]^{\frac{1}{2}}.$$  

(29)

For a weakly anisotropic material (10% anisotropy as defined by the ratio of maximum to minimum resistivities), the relative current density amplitude changes almost sinusoidally with electric field angle and the change in amplitude is relatively small (Figure 8a). For a strongly anisotropic material (200% anisotropy), the change in amplitude is much greater and is also less sinusoidal (note the different y-axis scales for 10% and 200% cases in Figure 8). Note that current density is deviated preferentially toward the direction of low resistivity in Figure 8a and that the mean current density exceeds what would be predicted by the arithmetic mean resistivity.

The azimuth angles of $E(\theta_E)$ and $\mathbf{J}(\theta_j)$ can be given as

$$\theta_E = \tan^{-1} \left( \frac{E_y}{E_x} \right),$$

(30)

and

$$\theta_j = \tan^{-1} \left( \frac{J_y}{J_x} \right) = \tan^{-1} \left( \frac{E_y \rho_x}{E_x \rho_y} \right).$$

(31)

Consequently, the angle between the electric field direction and that of the current density is given by

**Figure 7.** Resistivity (Qm) for oblate grains calculated using the HS conductive bound model (top), and the geometric path length effective resistivity method where the grains are aligned in the most resistive direction (middle) and the least resistive direction (bottom). Grain and fluid resistivities are as in Figure 4.

**Figure 8.** Current density (a), deviation angle between current density and electric field (b), and resistivity (c) as a function of electric field angle. Black lines indicate the values for a weakly anisotropic material where $\rho_{\text{min}} = 1$ Qm and $\rho_{\text{max}} = 1.1$ Qm (10% anisotropy). Grey lines indicate values for a strongly anisotropic material where $\rho_{\text{min}} = 1$ Qm and $\rho_{\text{max}} = 3$ Qm (200% anisotropy). In each plot, the y-axis scale for the weakly anisotropic material is given on the left and the strongly anisotropic material on the right.
\[ \delta \theta = \theta_E - \theta_J. \]  

For any anisotropic material, \( J \) is no longer parallel to \( E \), nor is it orthogonal to electric equipotentials when the electric field is not parallel to the minimum or maximum resistivity directions (Figure 8b). As anisotropy increases, the deviation angle increases. In a weakly anisotropic material, the deviation angle varies approximately sinusoidally with the electric field angle, but as the material becomes more anisotropic the relationship becomes more asymmetric.

Using the deviation angle and the current density, we can now calculate the change in resistivity as the current (electric field) angle rotates around the anisotropic medium (Figure 8c). Again, in the weakly anisotropic material, the observed resistivity varies almost sinusoidally with electric current angle, whereas that of the strongly anisotropic material varies more asymmetrically. The mean resistivity, averaged over all directions of electric field across the sample, is lower than the average of the maximum and minimum resistivity.

By combining the result with the maximum and minimum resistivities calculated using the geometric path length method, we can now calculate the resistivity of the effective medium at any current angle and grain aspect ratio. Figure 9 shows an example of the change in resistivity of an effective medium composed of aligned oblate grains at a porosity of 30%.

### Applications and limitations of the electrical model

The electrical model here describes an idealized material made from uncemented resistive ellipsoidal grains in a conductive fluid. The model is independent of grain size and, unlike Archie’s equation, allows the grains to have some conductivity. The model can therefore be applied to a range of sediment types, from clay to sand, although there are some limitations. First, all the inclusions in the model must have the same aspect ratio. This means the model is suitable for describing sediment that is composed predominantly of similar shaped grains, such as a clean sand, but not for describing a sediment where the grains are very differently shaped, such as a silty clay. A second limitation is that the model assumes all the grains have the same resistivity. Again, this means the model poorly represents a silty clay where the silt and the clay have very different resistivities.

### COMPARISON WITH LABORATORY DATA

In Figure 4, we compared the predictions of our geometric path length (GPL) electrical effective medium model to those of the Hashin-Shtrikman conductive bound for the isotropic case. Ideally, our model needs to be validated against physical observations on real two-phase systems. The most practical way to achieve this validation is to measure the electrical resistivity of sediment samples with known porosity and composition. Most real sediments are composed of many different types of grains each with their own physical properties and grain shape. Because our aim is to test a model in which the grains are treated as made up of only a single material, we used artificial sediment cores composed of spherical glass beads with known physical properties, saturated with pore water of known composition and resistivity. The use of spherical grains results in isotropic sediments. We did not attempt to make anisotropic materials.

### Method

The sediment specimens were prepared using a pluviation method. Pluviation provides reasonably homogeneous specimens (Rad and Tumay, 1985) and simulates the sediment fabric found in nature. It also produces the densest possible packing of mineral grains. The specimens were formed in a 50 cm length, 6.4 cm diameter plastic tube (of high electrical resistivity), with potential electrodes piercing the casing every 2 cm. Spherical glass beads ranging in size from 24 μm to 1000 μm were used to simulate sediment grains. The pore fluid was composed of de-aired, distilled water that was made into brine by adding common salt (25 g of NaCl per 1 liter of water). De-airing the brine ensured that the final sediment was fully saturated with brine and that no air was present. Once filled with glass beads and brine, the sediment tube was sealed using pistons at each end incorporating current electrodes. The sediment specimens were then placed in a temperature controlled room (set at 6°C for at least 24 hours prior to any measurements made to ensure that all the electrical measurements were made at the same temperatures.

Electrical resistivity was measured by passing a current of 10 ± 0.01 mA through the end electrodes of the sediment specimen at a frequency of 220 Hz (Figure 10). These current electrodes were made from disks of stainless steel mesh in order to distribute the current throughout the specimen.
rent as evenly as possible over the whole cross section of the specimen and so minimize edge effects. The potential difference was measured between neighboring potential electrodes along the length of the core to an accuracy of \pm 3.0 mV. Assuming that the current is distributed evenly through the cross section of the sediment filled tube, resistivity was calculated to an accuracy of \pm 3.6% using the equation,

$$\rho = \frac{VA}{DI} \quad (33)$$

where $D$ is the distance between the potential electrodes, $A$ is the cross-sectional area of the sediment cell (\( \sim 32.5 \, \text{cm}^2 \)), $V$ is the measured potential difference, and $I$ is the current.

Sediment density was measured at 0.5 cm intervals along the specimens using a multisensor sediment core logger (Gunn and Best, 1998). The density (gamma ray attenuation method) measurements are accurate to \( \pm 0.07 \, \text{g/cm}^3 \). Sediment porosity was derived from the measured bulk densities using the relationship,

$$\varphi = \frac{\chi_s - \chi}{\chi_s - \chi_f} \quad (34)$$

where $\chi$ is the sediment density, $\chi_s$ is the mineral density (2.50 \pm 0.05 g/cm$^3$ for the glass beads), and $\chi_f$ is the brine density (1.02 \pm 0.01 g/cm$^3$). The final porosity of the samples ranged from 30% to 47% (\( \pm 3.4\% \)). The resistivity of the pore fluid used in the laboratory samples was 0.36 \pm 0.01 \Omega m. A total of 77 independent measurements were made on a set of four artificial sediment cores of varying porosity (Ellis, 2008).

Results

Figure 11 shows the comparison between the laboratory data and the predictions of Archie’s equation (with various $m$ and $a$ coefficients), the HS conductive bound, and the geometric path-length effective resistivity model developed in this paper. The $m$ and $a$ coefficients used to calculate the first Archie curve are both equal to 1.0, values generally used when modeling straight cylindrical pore channels (Herrick and Kennedy, 1994). The second Archie curve is calculated using $m$ and $a$ coefficients of 1.25 and 1, respectively; these values are generally used to calculate the resistivity of unconsolidated sands and spherical glass beads (Archie, 1942; Wyllie and Slinger, 1952; Atkins and Smith, 1961; Jackson et al., 1978). The third and fourth Archie curves were created using $m$ and $a$ coefficients recommended in the Schlumberger log interpretation charts for soft sediments (Winsauer et al., 1952; Schlumberger, 1977). The final Archie curve was determined by allowing the $m$ and $a$ coefficients to vary until a best fit was found with the data.

Although the laboratory measurements cover only a relatively narrow range of porosities, this is not necessarily a great disadvantage. Any satisfactory model will predict resistivities that converge to the solid phase resistivity at 0% porosity and to the liquid phase resistivity at 100% porosity, apart from Archie’s equation when $a$ is not equal to one. The largest deviations between model predictions are likely to occur at mid-range porosities where our experimental data are located. Also, the range of porosities covered by our laboratory measurements is similar to that encountered in many natural sedimentary formations of interest.

An rms misfit was calculated between the predictions of each of the models and the laboratory data (Table 1). Both the HS conductive bound and the geometric path-length effective resistivity models predict the resistivities reasonably well. However, three of the five Archie curves do not go through any of the resistivity data points. The geometric path-length effective resistivity model has the lowest rms misfit to the laboratory data apart from the best fit Archie curve, improving on both the HS conductive bound and the second Archie equation curve for spherical glass beads ($m = 1.25, a = 1$). It is not surprising that the best fit Archie curve fits the data better than the geometric path-length effective resistivity model because this curve is essentially a regression in which the $m$ and $a$ coefficients are allowed to vary freely. However, such an approach has no predictive value. The other Archie’s curves are more appropriate for comparing experimental data to different Archie curves.

Figure 11. Comparison of artificial sediment experimental data (4 different specimens, frequency = 220 Hz, temperature = 6 °C) with model predictions from Archie’s equation with different cementation and the tortuosity constants, from the HS conductive bound model, and from the geometric path-length effective resistivity model. Grain and fluid resistivities are as in Figure 4.

### Table 1. Root-mean-square (rms) misfit values between the electrical model predictions (Archie, HS, and geometric path-length effective resistivity models) and the laboratory resistivity measurements on artificial sediments in Figure 11.

<table>
<thead>
<tr>
<th>Model</th>
<th>Resistivity rms misfit (( \Omega m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archie’s equation (where $m = 1, a = 1$)</td>
<td>0.451</td>
</tr>
<tr>
<td>Archie’s equation (where $m = 1.25, a = 1$)</td>
<td>0.183</td>
</tr>
<tr>
<td>Archie’s equation (where $m = 2.15, a = 0.62$)</td>
<td>0.765</td>
</tr>
<tr>
<td>Archie’s equation (where $m = 2, a = 0.81$)</td>
<td>1.007</td>
</tr>
<tr>
<td>Archie’s equation (where $m = 0.85, a = 1.7$)</td>
<td>0.104</td>
</tr>
<tr>
<td>HS conductive bound</td>
<td>0.134</td>
</tr>
<tr>
<td>Geometric path-length effective resistivity</td>
<td>0.123</td>
</tr>
</tbody>
</table>
son with the geometric path-length effective resistivity model because they use standard coefficients and therefore can be used to predict bulk resistivities in the absence of measurements.

Although the geometric path-length effective resistivity model does fit the data very well, it does not fit all the data points within the experimental errors. There may be several reasons for this. First, four separate sediment specimens were made in the laboratory. While every endeavor was made to ensure they were identical in terms of composition, there may have been small differences in the pore fluid salinity and impurities in the sediment grains. Such factors could have affected the final resistivity of the sediment but are outside the scope of the model. Another reason may be due to the range of glass bead sizes used in the different samples. A range of sizes was used to achieve a wide range of porosities (through smaller beads occupying the pore spaces between larger beads), while ensuring the composition of the sediment was the same between samples. If only one size had been used, the range of porosities would have been very small. While this approach appeared to work, it also may have altered the porosity versus resistivity trend between samples. Samples 1 and 4 contained the same mix of grain sizes whereas samples 2 and 3 had a different mix of grains sizes. Figure 11 shows that samples 1 and 4 appear to have the same trend, which differs from the other two samples. This observation indicates that the grain packing in the sediment may also affect the resistivity of the sediment, allowing sediments with the same porosity and pore fluid composition to have different resistivities. Finally, the porosity measurements are averages over the whole cross section of the sediment tube even though porosity may vary locally and hence deviate from the single value used to model each specimen.

CONCLUSION

We have developed a physical effective medium model for predicting the electrical resistivity of two-phase geological materials. The model assumes that the material is fully saturated and the fluid phase is fully interconnected and is the less resistive constituent. The geometrical arrangement of the solid inclusions is specified in terms of overall porosity and the aspect ratio of idealized ellipsoidal grains. The method allows both the fluid and solid phases to possess some conductivity, which is important for modeling sediments with grains. The method uses a geometric factor and mean path length approach to take account of the increase in path length taken by mobile charge carriers which are distributed preferentially in the fluid phase. The resistivity is dependent on the porosity and the aspect ratio of the grains, but is independent of grain size. Therefore, it can be applied to a wide range of sedimentary formations. However, in very low porosity formations, isolated porosity may be present and the model will start to break down.

The two-phase geometric path-length effective resistivity model was found to predict the observed resistivities in synthetive sediment specimens. It produced a better fit than either the HS conductive bound, or Archie’s equation with the m and a coefficients commonly used for soft formations or for loose glass bead samples.

The isotropic two-phase model was extended to the case of electrical anisotropy by aligning ellipsoidal grains with an aspect ratio of less than one. However, experimental data of electrical anisotropy in sediments and sedimentary rocks are needed to verify the anisotropic model predictions. Under a limited set of conditions, the geometric path-length effective resistivity model can also be extended to include a third, resistive phase. This could be applied to the prediction of water saturation from resistivity measurements on, for example, partially gas saturated, or gas hydrate-bearing, sediments. Again, more experimental data are needed to verify the model predictions.

ACKNOWLEDGMENTS

Michelle Ellis was supported by an Ocean Margins LINK PhD studentship from the United Kingdom Natural Environment Research Council. We thank four reviewers for their constructive comments.

REFERENCES


Downloaded 21 Feb 2012 to 216.198.85.26. Redistribution subject to SEG license or copyright; see Terms of Use at http://segdl.org/